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MIP Workshop 2025, Minneapolis

4th June 2025

GUROBI OPTIMIZATION

From Infeasibility to Feasibility

Improvement Heuristics to Find First Feasibles for MIPs

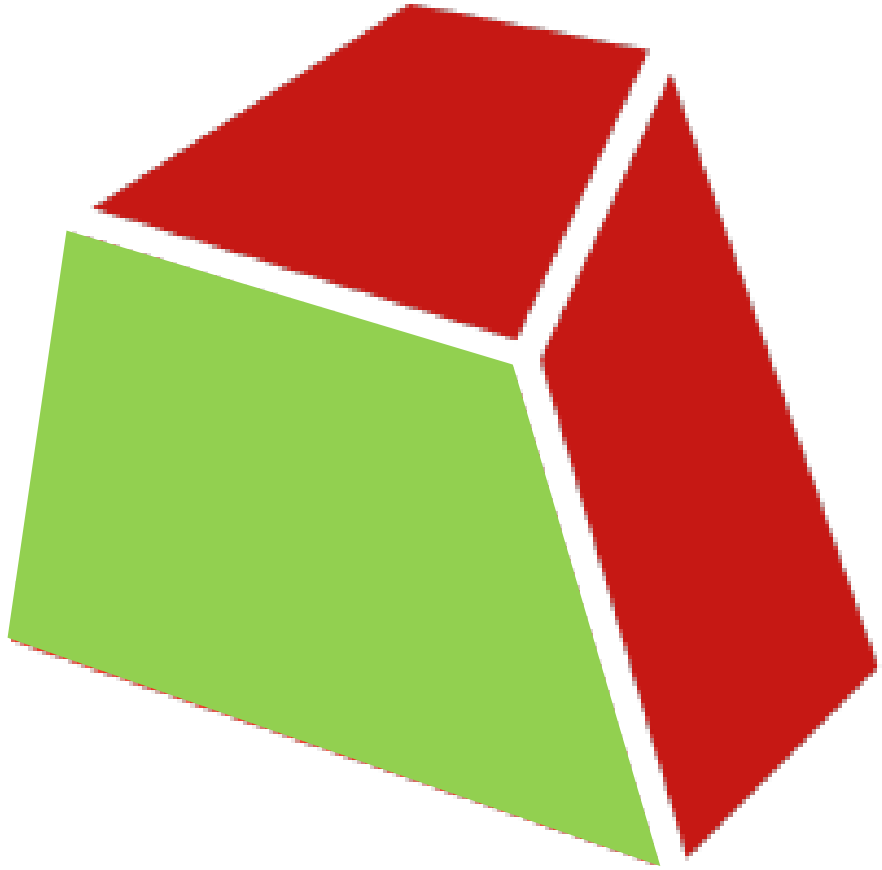
Finding Feasible Solutions in MIP

- User provided solution (“MIP Start”, callback)
- Integral LP solution
- Primal heuristics
  - Start heuristics
    - Many different approaches
    - Exploit certain structures (pure binary problems, network problems, scheduling, ...)
    - Search in the neighborhood of LP relaxation solutions
  - Improvement heuristics
    - 1-opt, 2-opt, other quick-and-dirty heuristics
    - Large neighborhood search

- See Berthold, Lodi, Salvagnin (2025): “Primal Heuristics in Integer Programming” for a comprehensive overview

Large Neighborhood Search in MIP

- Most common approach:
  - Consider the full problem
  - Fix some variables to certain values
  - Solve remaining problem as a sub-MIP



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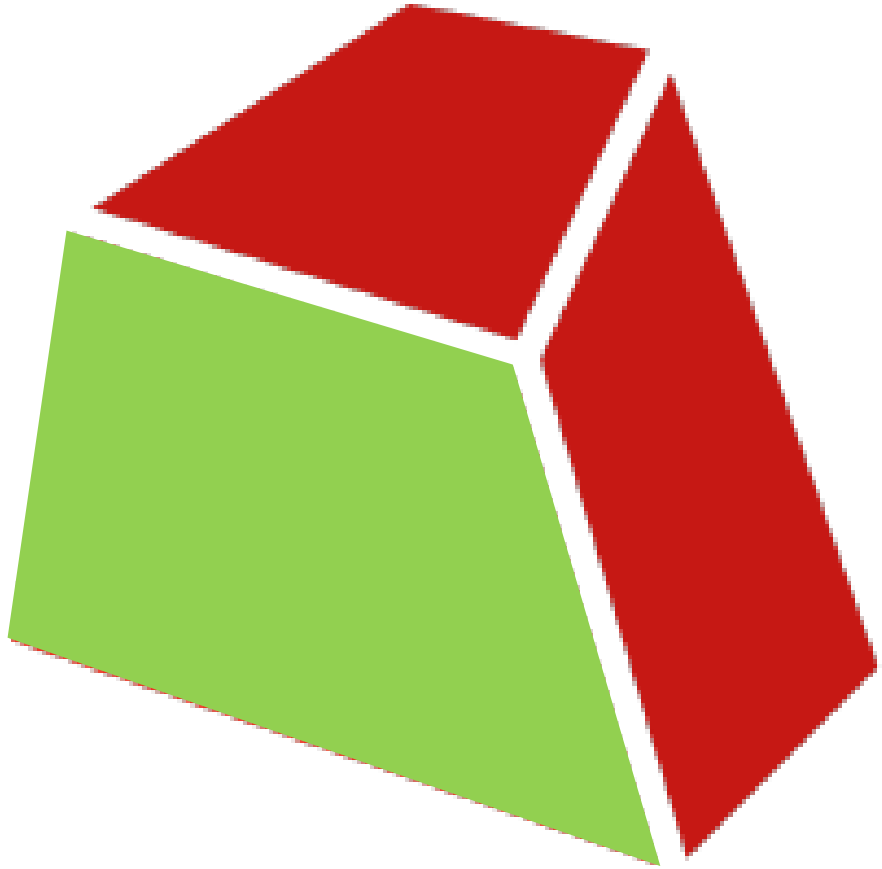
- "Target Heuristics" in Gurobi
  1. Specify an ordered list of variable fixings
     $$(x_i = f_i)_{i=1,\dots,K}$$
  2. For $i = 1, \dots, K$:
     - Fix $x_i := f_i$ and propagate
     - Backtrack last fixing if infeasible
     - Abort if infeasible again after backtracking
     - Break loop if sub-problem small enough
  3. Solve remaining sub-problem as MIP



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- Many different variants possible
- E.g., RINS: Sort integer variables by distance of incumbent to LP solution

- Fix-and-propagate: reduced to pure LP
- RINS: all variables fixed with incumbent equal to LP solution
- Can be dynamically adjusted

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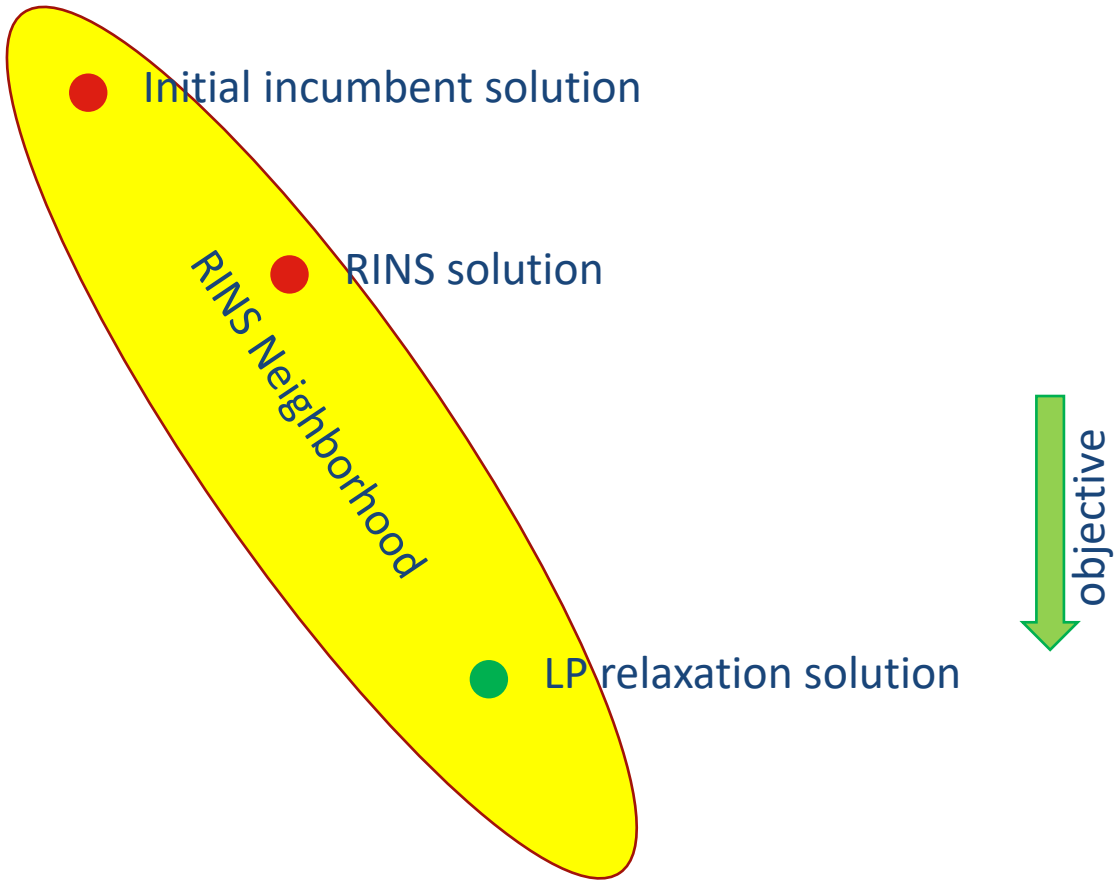
- "Target Heuristic" variants in Gurobi:
  - Lower bound heuristic
  - Variable hint heuristic
  - Partition heuristic
  - Rounding heuristics
  - Pseudo-cost fixing heuristic
  - Active domain heuristic
  - Big-M on-off heuristic
  - NLP heuristic
  - NoRel heuristic
  - RINS
  - RENS
  - MIP starts

Successive Improvements with RINS



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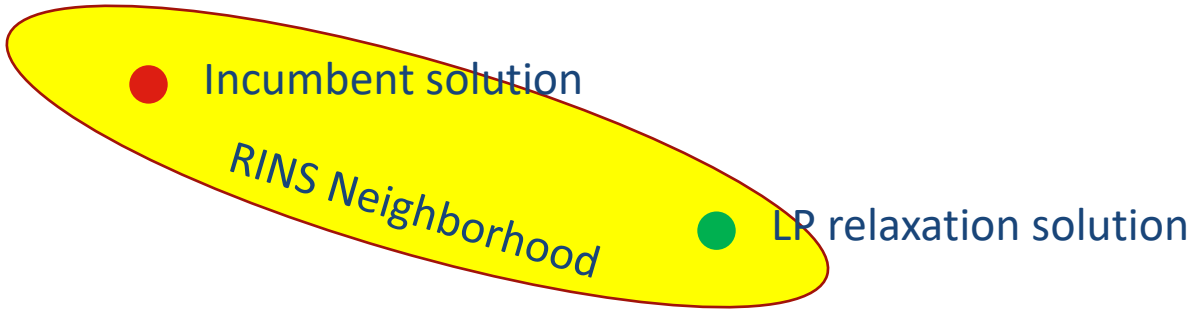

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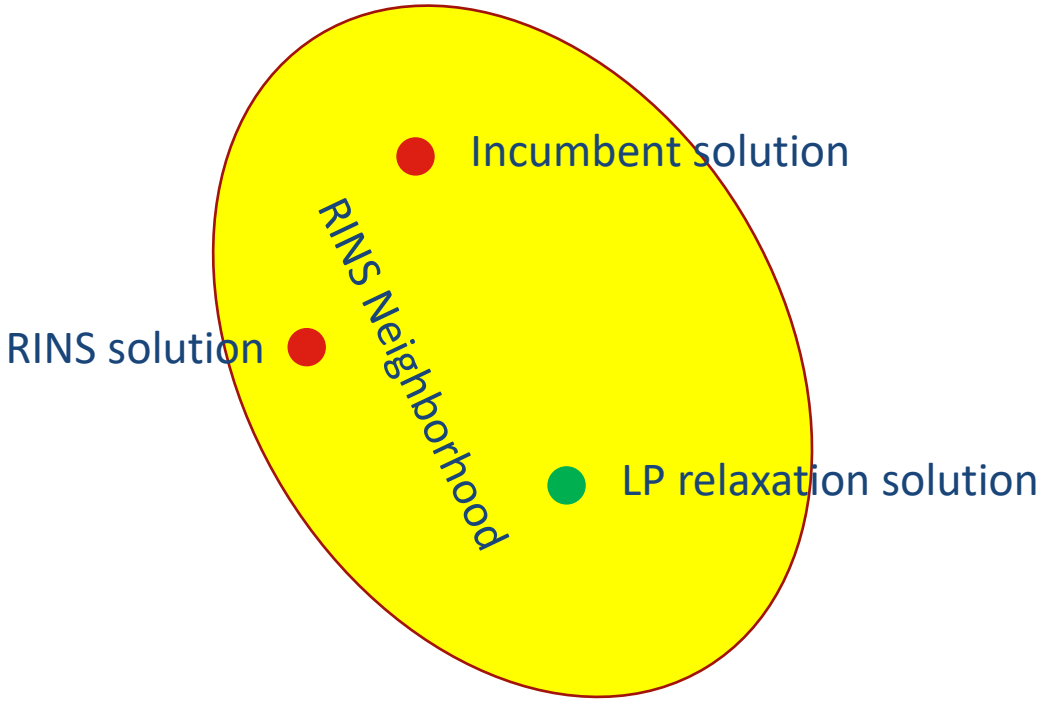

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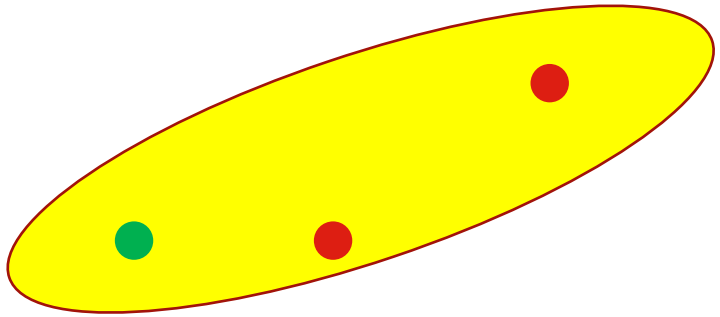

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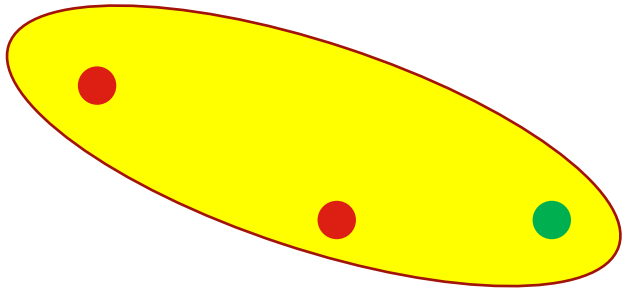

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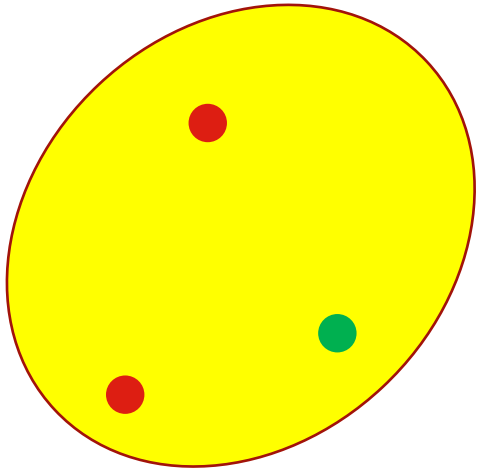
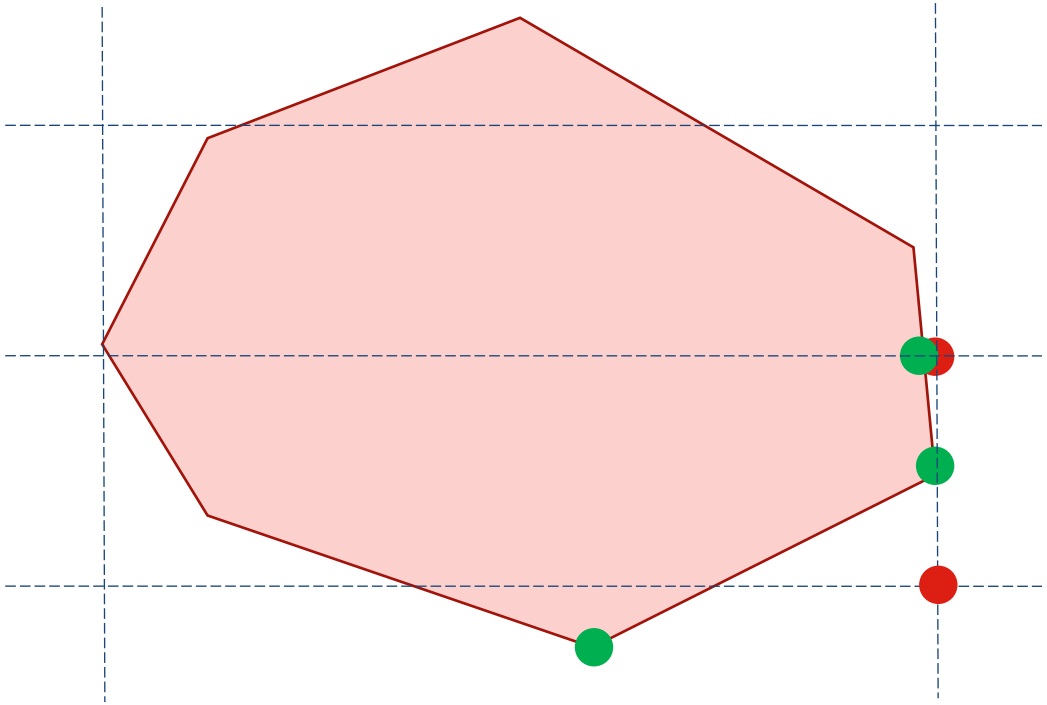


Successive Improvements with RINS

- RINS often very effective to improve solutions
  - Even if incumbent solution is pretty bad
- But: requires initial incumbent to get started

Feasibility Pump



- Fischetti, Glover, Lodi (2005): “The feasibility pump”
- Bertacco, Fischetti, Lodi (2007): “A feasibility pump heuristic for general mixed-integer problems”
- Fischetti, Salvagnin (2009): “Feasibility pump 2.0”

Feasibility Pump vs. RINS

GUROBI OPTIMIZATION

Feasibility Pump

- Solves sequence of LPs to inspect neighborhood
- Neighborhood defined by two points
  - Fractional LP-feasible solution
    - Current LP solution or FP iterate
  - Integral LP-infeasible solution
    - Rounded LP solution or FP iterate

RINS

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Something Else?

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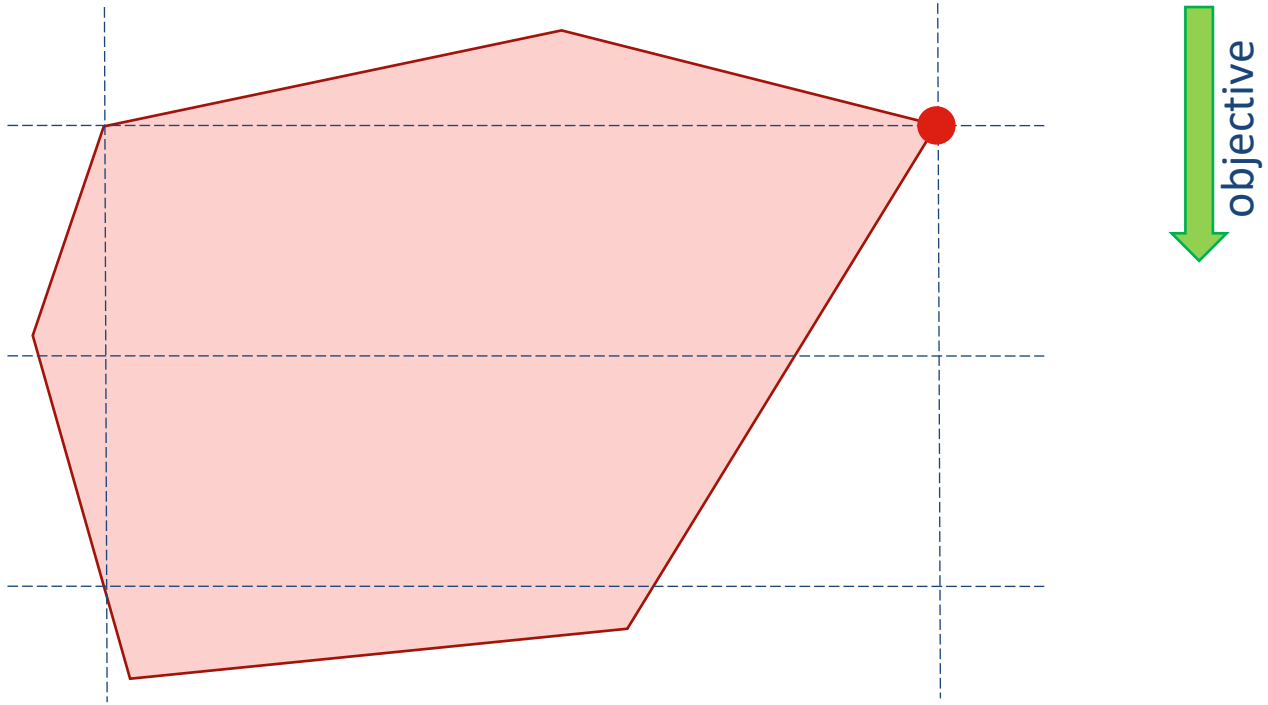
Infeasible Solution RINS

- Solves MIP to inspect neighborhood
- Neighborhood defined by two points
  - Fractional LP-feasible solution
    - Current LP solution
  - Integral LP-infeasible solution
    - Rounding of least fractional root LP solution

Where to Find Infeasible Solutions

- Incumbent solution from before presolve or before restart
  - MIP start, user callback solution, original model heuristic, before restart incumbent
  - Crushing of incumbent through presolve may be infeasible due to dual reductions



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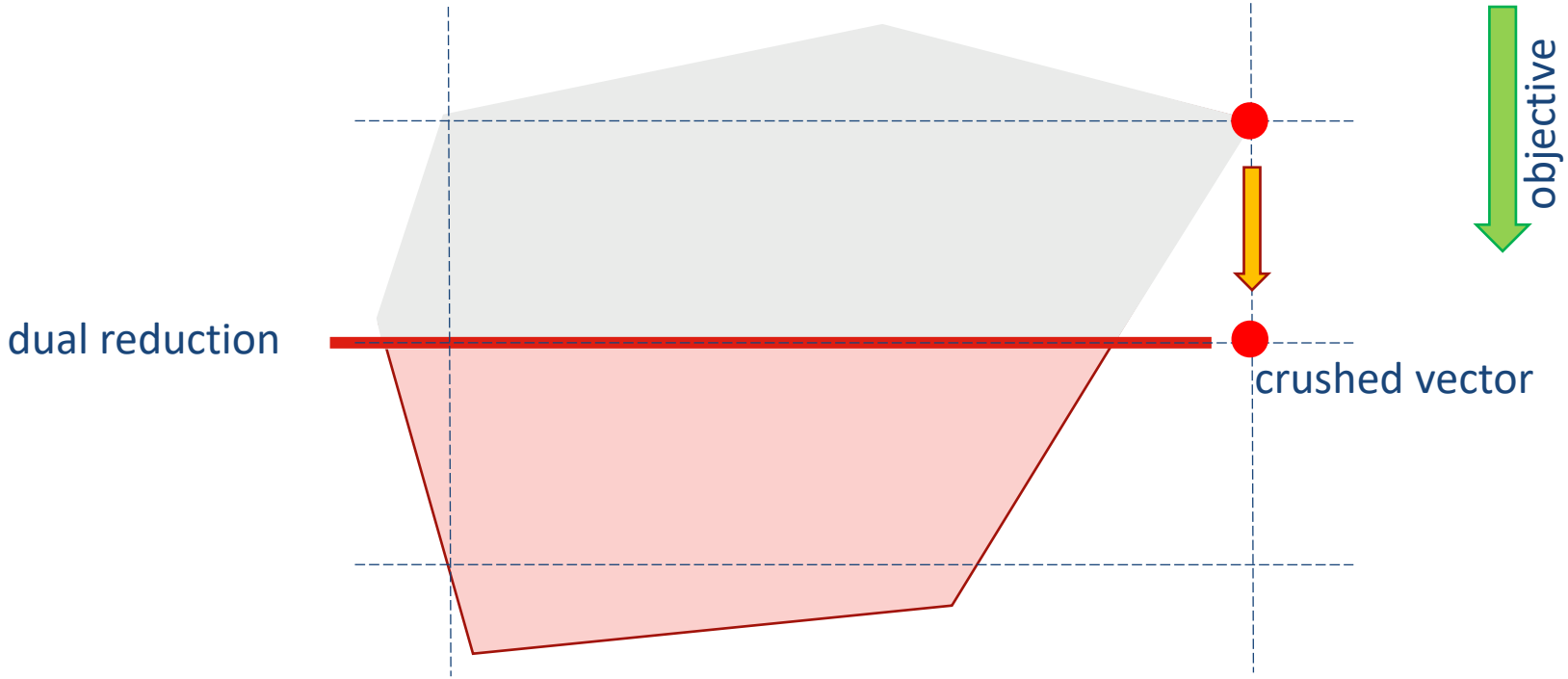

Where to Find Infeasible Solutions

- Incumbent solution from before presolve or before restart
  - MIP start, user callback solution, original model heuristic, before restart incumbent
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- Rounded root cut loop LP solution
  - Use LP solution with smallest number of integer infeasibilities
  - Only at root node, not at local nodes
- Infeasible solution from before restart
  - Crushing of infeasible solution through presolve
- “Feasible” solutions found in sub-MIP heuristics that violate feasibility tolerances
- Infeasible solution found in any sub-MIP heuristic
  - E.g., rounded root node LP solution of a RINS sub-MIP

How to Use Infeasible Solutions

- NoRel heuristic
  - Populate initial NoRel solution pool with infeasible solution
- Partition heuristic
  - Use infeasible solution as fixing values for target heuristic
- All variants of RINS heuristics

- Additional ideas (not yet implemented)
  - Local branching
  - Fixing target for fix-and-propagate and fix-and-dive heuristics
  - Guided dives for node selection
  - Cut filtering

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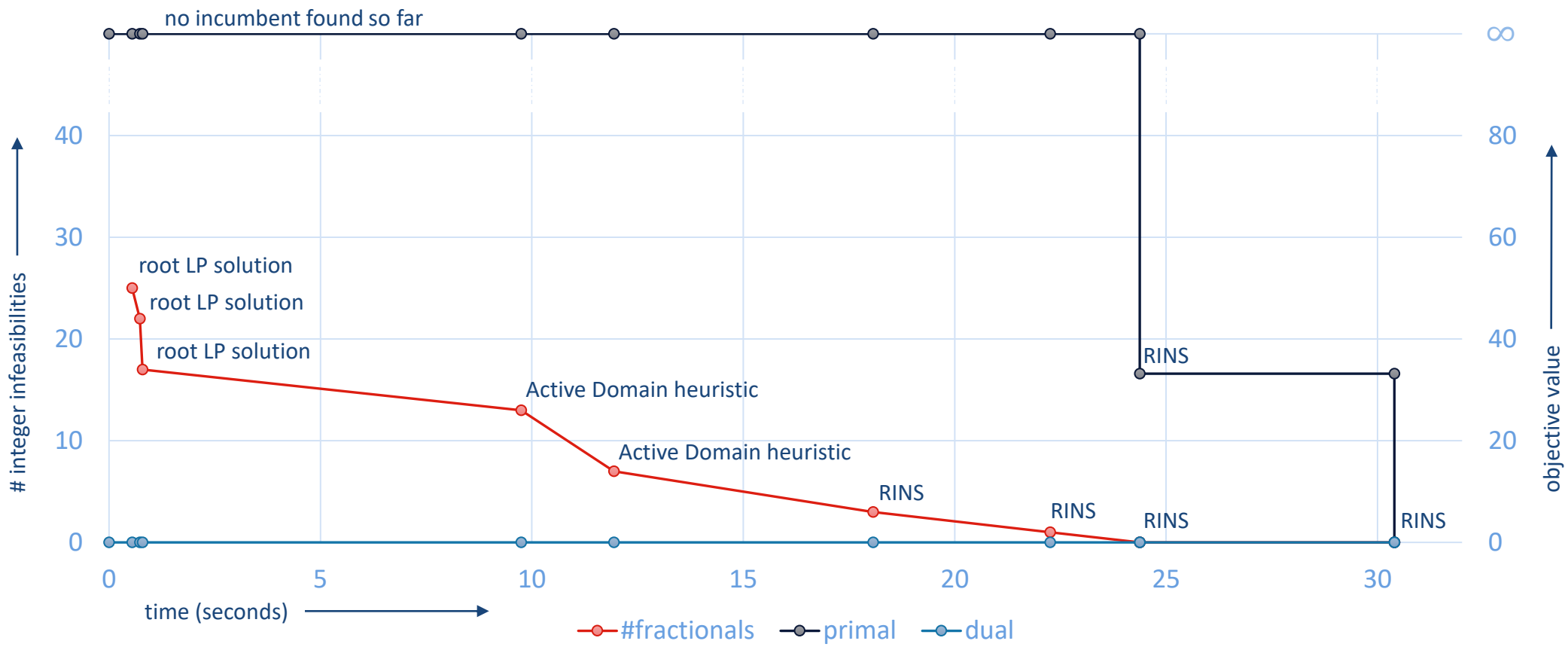
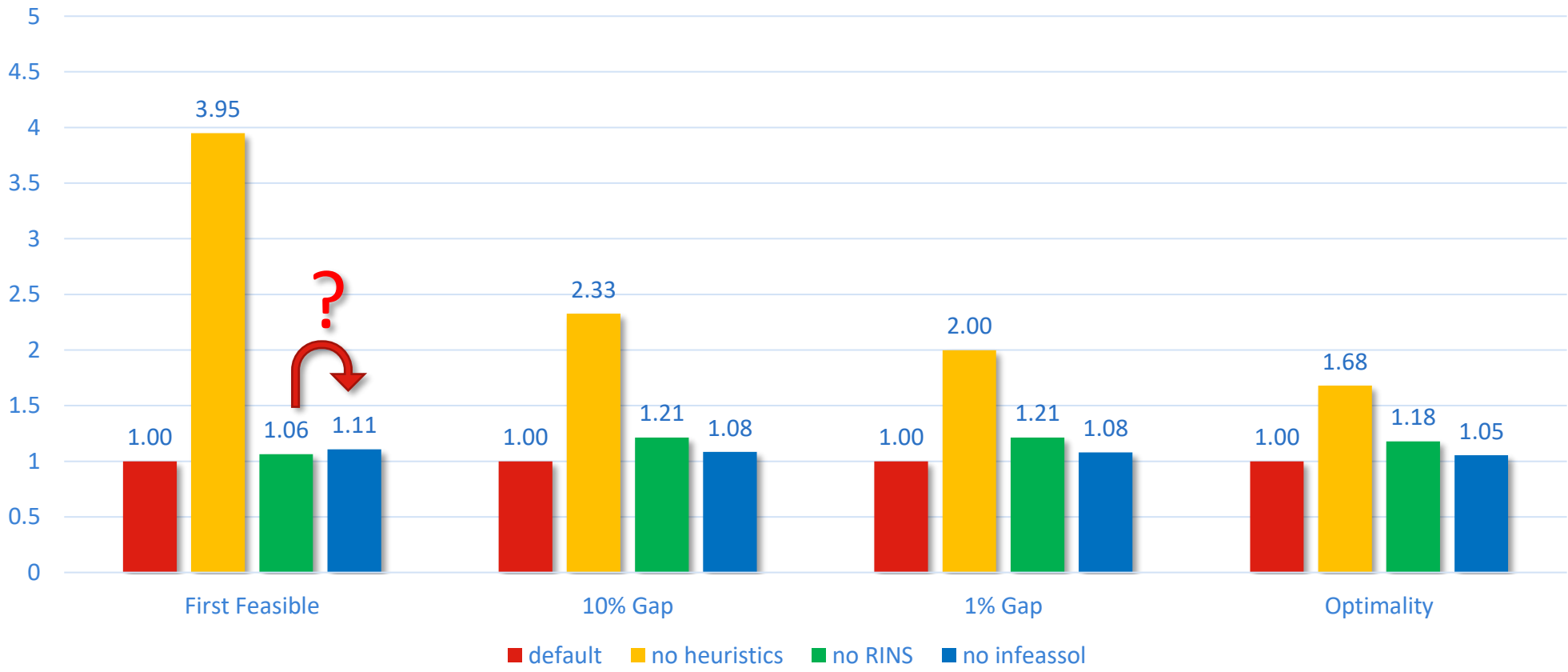


Performance Impact

Solvable Models (t_{max} > 100 sec)

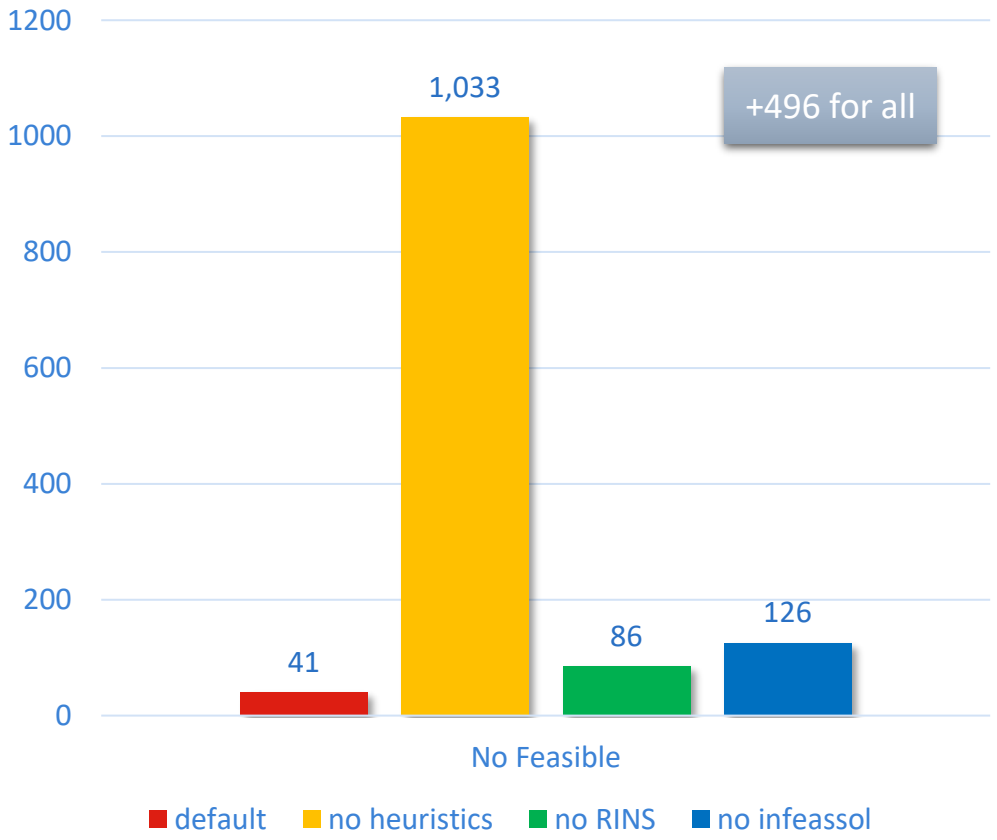


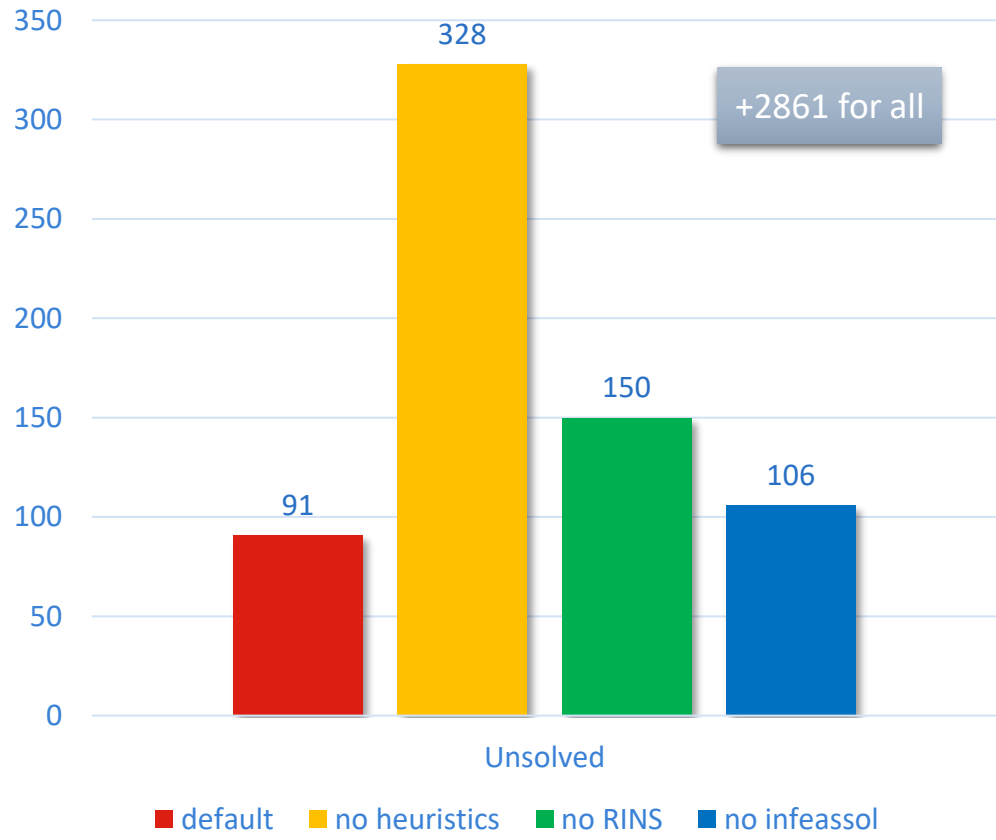
Time limit: 10000 sec.
Intel Xeon CPU E3-1240 v5 @ 3.50GHz
4 cores, 8 hyper-threads
32 GB RAM

Test set has 8611 models:
- 102 discarded due to inconsistent answers/fails
- 2861 discarded that none of the versions can solve
- speed-up measured on >100s bracket: 1658 models

Performance Impact

All Models





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- numbers counted on all models: 8509 models

We are Hiring Technical Staff at Gurobi!

- Optimization Engineer (USA Remote)
- Optimization Strategist (USA Remote)
- Optimization Strategist (EU Remote)

- Senior MIP Developer (Global Remote)

- Technical Account Manager (USA Remote)
- Technical Account Manager (EU Remote)
- Technical Account Manager (APJ Remote)
- Technical Account Manager (Australia Remote)

- See https://www.gurobi.com/company/careers/

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Thank You!

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